CONNECTICUT STATE ASSOCIATION OF MATH LEAGUES
State Math Match, 2025

Please write your answers on the answer sheet provided.

Round 1: Arithmetic and Number Theory

1-1 121% of $\left(\frac{1}{3}+\frac{2}{5}\right)^{-3}$ is $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a+b$.

1-2 Find the smallest positive integer n such that $n! \cdot (n-1)! \cdot (n-2)! \cdot \cdots \cdot 1!$ is divisible by 2^{56}.

1-3 Two ordinary, fair dice are thrown repeatedly. On each throw, the "score" is the sum of the numbers on the two uppermost faces. The probability that the first score of 4 is obtained earlier in the sequence of scores than the first score of 7 and the first score of 7 is obtained earlier than the first score of 5 is $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a+b$.

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Round 2: Algebra I

2-1 For any real, positive value of x, $\sqrt{x \cdot \sqrt[3]{x \cdot \sqrt[4]{x}}} = x^{p/q}$, where p and q are relatively prime positive integers. Find $p + q$.

2-2 Find the sum of the squares of all real values of x such that

$$\frac{|x|}{1 + |x|} = \frac{6 + x}{1 - x}$$

2-3 Find the number of integer values of n, $1 \leq n \leq 2025$, such that the following inequality has exactly two integer solutions.

$$\left|\frac{nx}{2025} - 1\right| < \frac{n}{2025}$$

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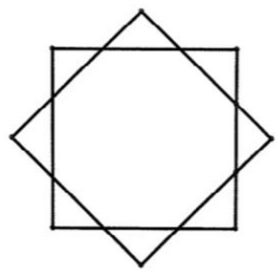
Round 3: Geometry

Diagrams are not drawn to scale.

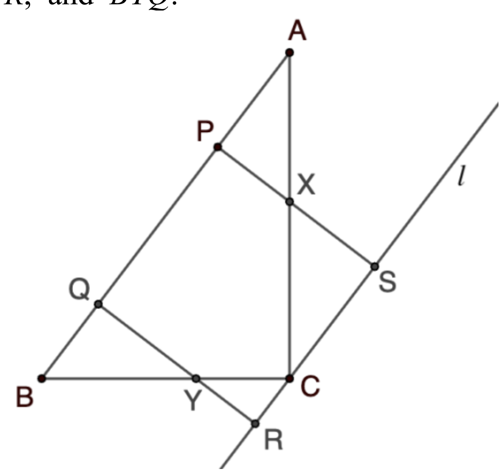
3-1 A pyramid has a square base with side length 10, and the height of the pyramid is 12. When the base of the pyramid is placed on a horizontal surface, the vertex of the pyramid lies directly above the center of the base. Find the total surface area of the pyramid, including the base.

3-2 As shown in the diagram below, two congruent squares overlap in such a way that their intersection points form a regular octagon.



If the side length of the original squares is 2, then the area of the octagon is $a\sqrt{b} + (-1)^n c$, where a, b, and c are positive integers, n is 0 or 1, and b is not divisible by the square of any prime number. Find $a + b + c + n$.

3-3 In right triangle ABC with right angle C, line l is drawn through C and is parallel to $\overline{AB}$, as shown in the diagram below. Points P and Q lie on $\overline{AB}$ with P between A and Q, and points R and S lie on l with C between R and S such that $PQRS$ is a square. Let $\overline{PS}$ intersect $\overline{AC}$ in X, and let $\overline{QR}$ intersect $\overline{BC}$ in Y. The inradius of triangle ABC is 10, and the area of square $PQRS$ is 576. Find the sum of the inradii of triangles AXP, CXS, CYR, and BYQ.



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Round 4: Algebra II

4-1 The first term of an infinite geometric series is 8, and the sum to infinity of the series is 12. The fourth term of the series is $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a+b$.

4-2 Suppose that $4^{1/x} - 8^{1/y} = 0$ and $\log_2 x - \log_4 y = 0$. Then $x + y = \frac{p}{q}$, where p and q are relatively prime positive integers. Find $p+q$.

4-3 Let $P(x)$ be a fifth-degree polynomial with integer coefficients such that the equation $P(x) = 0$ has at least one integer solution. Suppose that $P(2) = 13$ and $P(10) = 5$. Find the value of x that *must* satisfy the equation $P(x) = 0$.

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Round 5: Analytic Geometry

5-1 Parabola 1 has equation $y = x^2 - 6x + 10$ and Parabola 2 has equation $y = 2x^2 + 3x$. The parabolas intersect at point P in the first quadrant. Let the distance of P from the vertex of Parabola 1 be d. Find d^2.

5-2 The center of a circle lies on the y-axis, and the circle passes through the points $(7, -1)$ and $(-3, -3)$. The circle intersects the positive x-axis at $(h, 0)$. Find h^2.

5-3 The line $y = mx + b$ is tangent to the curve $y = x^4 + 4x^3 - 16x^2 + 6x - 5$ at two points. Find $m - b$.

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Round 6: Trigonometry and Complex Numbers

6-1 Let $z = 1 + 3i$. Then $\left(3z + \frac{10}{z}\right)^2 = a + bi$, where a and b are real. Find $|a| + |b|$.

6-2 Suppose that

$$\tan\theta = \frac{5\cos\theta}{3\sin\theta - 2\cos\theta}$$

Then the sum of the squares of the possible values of $\tan\theta$ is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

6-3 Let $S = \sin^2 4° + \sin^2 8° + \sin^2 12° + \cdots + \sin^2 176°$.
Then $S = \frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

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Team Round

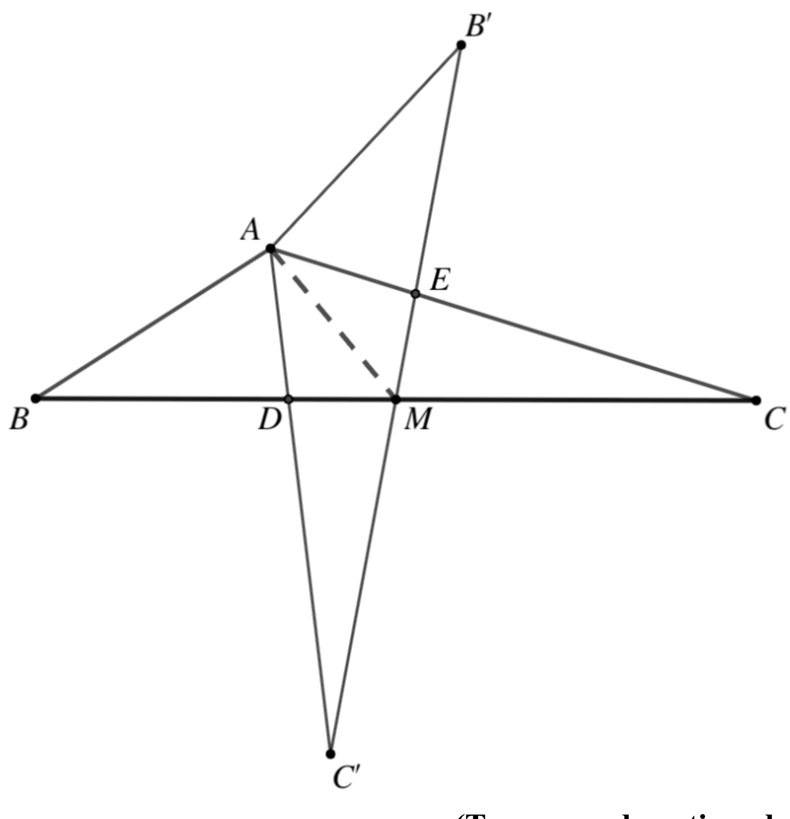
Diagrams are not drawn to scale.

T-1 Suppose that $\frac{1}{n}$ is added to $\frac{1}{100}$ and the sum is expressed as a fully reduced fraction. For how many values of n selected from the set $\{1, 2, 3, \ldots, 99\}$ is the denominator of the sum equal to $100n$?

T-2 The Twins-R-Us day care center has 35 sets of twins enrolled, and these are the only children enrolled in the program. Of the children enrolled in the program, 38 are boys, and there are four more sets of girl-girl twins than of girl-boy twins. How many sets of boy-boy twins are enrolled?

T-3 [Terminology: The "reflection of point P in line l" is a point P' (called the image of P) such that l is the perpendicular bisector of the line segment $\overline{PP'}$. The reflection of a set of points is the set of their images.]

Triangle ABC is reflected in its median $\overline{AM}$ (extended), as shown in the diagram below. The images of points B and C are B' and C', respectively. Let D be the point of intersection of $\overline{BC}$ and $\overline{AC'}$ and let E be the point of intersection of $\overline{AC}$ and $\overline{B'C'}$. Suppose that $AE = 6$, $EC = 12$, and $BD = 10$. Then $AB = p\sqrt{q}$, where p and q are positive integers and q is not divisible by the square of any prime number. Find $p + q$.



(Team round continued on next page)

T-4 Let

$$(x^3 + ax^2 + 2x - 5)^{19}(x^2 + bx - 41)^8(x^4 - x^3 + x - 7)^6$$
$$= x^{97} + 391x^{96} + c_{95}x^{95} + c_{94}x^{94} + \cdots + c_1x + c_0$$

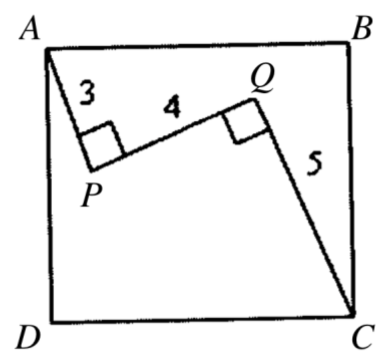
be an identity, where $a, b, c_{95}, c_{94}, \ldots, c_1, c_0$ are integers, and $a + b < 10$. Find the smallest possible value for a.

T-5 Ellipse 1 lies in the first quadrant and is tangent to both axes; its major axis is parallel to the *y*-axis and its minor axis is parallel to the *x*-axis; the length of its semi-major axis is 2 and the length of its semi-minor axis is 1. Ellipse 2 has the same center as Ellipse 1, has the same eccentricity as Ellipse 1 (meaning that it is similar to Ellipse 1), and is internally tangent to Ellipse 1. The major axis of Ellipse 2 is perpendicular to the major axis of Ellipse 1. If the equation of Ellipse 2 is written as

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

then $a + b + h + k = \frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

T-6 In the diagram, *ABCD* is a square, $\overline{AP} \perp \overline{PQ}$, $\overline{PQ} \perp \overline{QC}$, $AP = 3$, $PQ = 4$, and $QC = 5$. Find the area of square *ABCD*.



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Answers

Round 1

1-1	179
1-2	12
1-3	74

Round 2

2-1	41
2-2	9
2-3	2010

Round 3

3-1	360
3-2	19
3-3	14

Round 4

4-1	35
4-2	19
4-3	15

Round 5

5-1	20
5-2	66
5-3	151

Round 6

6-1	68
6-2	43
6-3	47

Team Round

T-1	40
T-2	15
T-3	11
T-4	31
T-5	11
T-6	40